Constraint on the Mass of Primordial Black Holes from the Cosmological Constant

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In a recent propposal, the cosmological constant has been considered as as a new thermodynamical variable and its change is related to the radiation produced by black holes. Using this consideration and by modelling the primordial black holes as Schwarzschild-de Sitter holes,we have constrained the total mass of primordial black holes evaporated by now, giving an estimate of the order of $1.624 \times 10^{24} M_{\odot}$.

In the early universe, the great compression associated with the Big Bang could have formed black holes with different masses. These are known as Primordial Black Holes (PBH) and are an important because they could provide some proofs about these early stages of the universe, the gravitational collapse, the quantum evaporation and about quantum gravity. The number of PBH is unknown but some estimated densities have been calculated [1]. In particular, PBH constraints can be obtained by considering a varying gravitational "constant" G, that can be different at early times. The simplest example is the Brans-Dicke theory, in which G is associated with a scalar field that can be non-homogeneus. Therefore, the cosmological consequences of the variation of G depend on the PBH evaporation and how evolves the value of G near the black hole[2, 3].

On the other hand, the cosmological constant has an important role in the evolution of the universe, including the early universe and the Big Bang. Recently, some authors claimed that the cosmological constant can be promoted to a thermodynamical state variable [4, 5]. Then, the first law in differential form, is modified to be

$$dM = TdS + \Omega dJ + \Phi dQ + \Theta d\Lambda, \tag{1}$$

where Θ is the generalized volume conjugate to the cosmological constant Λ. The integral mass formula is generalized to

$$\frac{n-3}{n-2}M = TS + \Omega J + \frac{n-3}{n-2}\Phi Q + \frac{1}{n-2}\Theta\Lambda, \tag{2}$$

where n is the dimension of the spacetime. Using this interpretation, it is argued that the decrease of vacuum energy, represented by the cosmological constant, is equal to the decreasing of entropy inside the black hole horizon. Therefore, for an external observer, this process is seen as if the vacuum energy is transformed quantum mechanically to the energy of radiation of the black hole. This effect attributes the Hawking radiation to the varying cosmological constant.

In this paper, we will consider the inverse process, i.e. that the Hawking radiation produced by the evaporation of a black hole may produce a variable cosmological constant. In particular, we will consider the observational value of the cosmological constant today to constrain the mass of the PBH produced in the early universe using this process.

I. PRIMORDIAL BLACK HOLES

In the early universe, black holes with a wide range of masses could have formed as a consecuence of the compression associated with the Big Bang. These objects are known as Primordial Black Holes (PBH) and their horizon mass M depends on their formation epoch. If t is the time after the Big Bang, the comparison between the cosmological density and the density of a black hole shows that the mass of a PBH formed at time t is given by[1]

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$$M(t) \approx \frac{c^3 t}{G} \approx 10^{15}\left(\frac{t}{10^{-23}s}\right) g. \tag{3}$$

At early times, the mass of a PBH is really small. For example, at Planck's time, $t = 10^{-43}s$, the mass of a PBH is just $M = 10^{-5}g$. These small masses indicate that thermodynamical effects need to be considered because, as is well known, the Hawking temperature depends on the black hole's mass. Therefore, a PBH has a temperature of the order

$$T = \frac{\hbar c^3}{8\pi G k_B M} \approx 10^{-7}\left(\frac{M}{M_\odot}\right)^{-1} K, \tag{4}$$

i.e. that the smaller the mass of the black hole, the higher the temperature. Since the energy of the radiation is supplied by the black hole's mass, it is expected that, after some time, the black hole evaporates. The time scale of this process of evaporation depends on M. For a PBH, it is given by

$$\tau(M) \approx \frac{\hbar c^4}{G^2 M^3} \approx 10^{64}\left(\frac{M}{M_\odot}\right)^3 \text{years}. \tag{5}$$

This timescale shows that black holes with masses smaller than $10^{15}g$ have evaporated completely by now and equation (3) implies that these PBH formed before $t = 10^{-23}s$.

II. SCHWARZSCHILD-DE SITTER BLACK HOLE

The Scharzschild-de Sitter (SdS) space time in its static form is given by the line element

$$ds^2 = f(r)\, dt^2 - \frac{1}{f(r)} dr^2 - r^2 d\Omega^2 \tag{6}$$

where

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2. \tag{7}$$

This is a spherically symmetric spacetime with two parameters: M, the mass of the black hole and $\Lambda > 0$, the cosmological constant. The horizons of this solution are defined by the equation

$$1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2 = 0, \tag{8}$$

that corresponds to a third order polynomial. The solutions of this equation are parameterized by trigonometric functions and their inverses [6], as

$$r_1 = -\frac{2}{\sqrt{\Lambda}} \cos\left(\frac{1}{3}\cos^{-1}\left(3M\sqrt{\Lambda}\right)\right) \tag{9}$$

$$r_2 = -\frac{2}{\sqrt{\Lambda}} \cos\left(\frac{1}{3}\cos^{-1}\left(3M\sqrt{\Lambda}\right) + 2\pi\right) \tag{10}$$

$$r_3 = -\frac{2}{\sqrt{\Lambda}} \cos\left(\frac{1}{3}\cos^{-1}\left(3M\sqrt{\Lambda}\right) + 4\pi\right). \tag{11}$$

The involved trigonometric functions in these solutions forbid any result with $3M\sqrt{\Lambda} > 1$, and gives a maximum mass for the black hole,

$$M_{max} = \frac{1}{3\sqrt{\Lambda}}. \tag{12}$$

Note that the first solution r_1 has a maximum value

$$r_1 \left(M_{max} \right) = -\frac{2}{\sqrt{\Lambda}}, \tag{13}$$

that shows how this root is always negative. Thus, the SdS black hole has only two physical horizons r_2 and r_3. These roots can be expanded retaining corrections up to third order in $M\sqrt{\Lambda}$, [6], as

$$r_2 \approx \sqrt{\frac{3}{\Lambda}} - M \tag{14}$$

$$r_3 \approx 2M \left(1 + \frac{4}{3} M^3 \Lambda^{3/2} \right). \tag{15}$$

When considering the cosmological constant as a new thermodynamical state variable, the integral form of the first law for this kind of black hole has the form of equation (2) where the new term, $\Theta\Lambda$, has dimensions of energy. Since the cosmological constant is related with the vacuum energy density,

$$\rho_{vac} = \frac{\Lambda}{8\pi}, \tag{16}$$

the function Θ is interpreted as a generalized volume. For a Schwarzschild-de Sitter black hole, Θ is given by[4]

$$\Theta = \left(\frac{\partial M}{\partial \Lambda} \right)_S = -\frac{r_H^3}{6}. \tag{17}$$

where r_H is the radius of the event horizon. As can be seen, Θ corresponds to the volume of the region occupied by the black hole, with a pre-factor. Thus, the thermodynamical contribution $\Theta\Lambda$ can be written as

$$\Theta\Lambda = -\frac{4\pi r_H^3}{3} \left(\frac{\Lambda}{8\pi} \right), \tag{18}$$

that is exactly the product of the vacuum energy density and the volume inside the event horizon of the black hole. Therefore, the first law of thermodynamics is now

$$dM = TdS + \Theta d\Lambda. \tag{19}$$

III. CONSTRAINT ON THE PBH MASS BY THE COSMOLOGICAL CONSTANT VALUE

If we consider that PBH can be modelled as Schwarzschild-de Sitter black holes, and we suppose that their evaporation process does not affect the entropy, i.e. $dS = 0$, the firts law (19) gives

$$dM = \Theta d\Lambda = -\frac{4\pi r_H^3}{3} d \left(\frac{\Lambda}{8\pi} \right), \tag{20}$$

that can be interpreted by saying that the decrease of mass by Hawking evaporation produces a change of the cosmological constant. The ubication of the event horizon for the Schwarzschild-de Sitter black hole is given as the larger of the roots r_2 and r_3. By comparing the observational value of the cosmological constant and the maximum

value of the masses of PBH that have evaporated by now, we conclude that the event horizon corresponds to the r_2 root, that is approximated by

$$r_H = r_2 \approx \sqrt{\frac{3}{\Lambda}}. \tag{21}$$

Thus, the first law becomes

$$dM = -4\pi\sqrt{3}\Lambda^{-3/2} d\left(\frac{\Lambda}{8\pi}\right). \tag{22}$$

This equation can be integrated to obtain

$$M - M_0 = \Delta M_{PBH} = \sqrt{\frac{3}{8\pi}}\sqrt{8\pi}\Lambda^{-1/2},$$

where M_0 is an integration constant that can be interpreted as the initial mass of the black hole. Then, the vacuum energy density can be expresed as

$$\rho_{vac} = \frac{\Lambda}{8\pi} = \frac{3}{8\pi\left(\Delta M_{PBH}\right)^2}, \tag{23}$$

where ΔM_{PBH} represents the mass of primordial black holes that is completely evaporated at some epoch. Recent cosmological observations imply that the cosmological constant has the limit value[7]

$$\left|\rho_{vac}^{obs}\right| \leq 2 \times 10^{-10}\frac{erg}{cm^3}. \tag{24}$$

Therefore, the estimate of ρ_{vac} constrains the PBH mass that is completely evaporated by now to

$$\Delta M_{PBH} \geq \left[\frac{1}{144\pi} \times 10^{10}\frac{cm^3}{erg}\right]^{1/2} \tag{25}$$

$$\Delta M_{PBH} \geq 1.148 \times 10^{57} gr. \tag{26}$$

In solar masses, this result is,approximately,

$$\Delta M_{PBH} \gtrsim 1.624 \times 10^{24} M_{\odot}. \tag{27}$$

IV. CONCLUSION

When considering the cosmological constant as a new thermodynamical variable, its change is related to the radiation produced by black holes. By modelling the primordial black holes as Schwarzschild-de Sitter holes and using the observational value of the cosmological constant today we have constrained the total mass of primordial black holes evaporated by now to be of the order of $1.624 \times 10^{24} M_{\odot}$.

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